# PART VI

## Radiated Energy and the Quantum

All forms of electrodynamics state that energy is radiated out in all directions from an electric charge in accelerated motion. This radiated energy falls to zero when the acceleration is zero, and when there is uniform motion in a straight line.

A description of a highly accelerated motion of an electron has been given in the preceding pages as it moves out from the nucleus and back. (see eq. 77). It is possible to calculate how much energy is lost by radiation during one complete excursion of the electron out from the nucleus and back to it.

The rate of radiation of energy at any instant by an accelerated point charge has been determined by Lienard and later verified by G. A. Schott,* as follows

$$\dot{R} = \frac{2}{3} c e^2 \left[ \frac{\beta^4}{\rho^2 (1-\beta^2)^2} + \frac{\frac{1}{c^4}\left(\frac{d^2 r}{d t^2}\right)^2}{(1-\beta^2)^3} \right] \qquad (84)$$

where $\rho$ is the radius of curvature of the path, $\frac{d^2 r}{d t^2}$ the acceleration and $\beta c$ the velocity of the point.

*G. A. Schott, Electromagnetic Radiation, pp 110 and 251.
Lienard, L'Eclairage, electrique, July 1898.
Wiechert, Archives Neerlandaise, 1900, p 549.

In the case under discussion, where the electron moves in a straight line only so that the radius of curvature of the path is infinite, the first term of equation (84) vanishes, giving the rate of radiation of energy as follows,

$$\mathcal{R} = \frac{2e^2}{3c^3}\frac{\left(\frac{d^2r}{dt^2}\right)}{(1-\beta^2)^3} \qquad (85)$$

Writing for beta its equivalent $dr/c\,dt$, the total energy radiated away from the electron during one whole excursion out and back to the atom is

$$E = \int_{t=0}^{t=\infty} \mathcal{R}\,dt = \frac{2e^2}{3c^3}\int_{t=0}^{t=\infty}\left[1-\frac{1}{c^2}\left(\frac{dr}{dt}\right)^2\right]^{-3}\left(\frac{d^2r}{dt^2}\right)^2 dt \qquad (86)$$

By expanding the binomial into infinite series, equation (86) is equivalent to the following,

$$E = \frac{2e^2}{3c^3}\int_{t=0}^{t=\infty}\left[1+\frac{3}{c^2}\left(\frac{dr}{dt}\right)^2+\frac{6}{c^4}\left(\frac{dr}{dt}\right)^4+\frac{10}{c^6}\left(\frac{dr}{dt}\right)^6+\cdots\right]\left(\frac{d^2r}{dt^2}\right)^2 dt \qquad (87)$$

Each term of the series (87) may be integrated separately by making use of the expressions for the acceleration in (74) and the velocity in (76) above given. After integrating between the



limits of time zero to infinity, the final result is

$$E = \frac{\pi e^2 k_1^2}{2c^3}\left[1+\frac{23}{90}\left(\frac{k_1}{c}\right)^2+\cdots\right]\nu \qquad (88)$$

The next term of this series will be $(k_1/c)^4$ with a different numerical coefficient, and so on, the powers of $k_1/c$ increasing by 2 from term to term. But $k_1/2$ is the initial velocity of projection of the electron from the atom at the mo- of severing contact with the positive charge, which has been assumed to be close to the velocity of light. Therefore the sum of the series in the bracket of (88) is a constant, and the factor before the bracket is also constant. Denoting this constant coefficient of the frequency $\nu$ by H, equation (88) is the equivalent of

$$E = H\nu \qquad (89)$$

This equation would be the exact equivalent of Planck's quantum formula

$$E = h\nu \qquad (90)$$

if it can be shown that the constant H in (89) is the same as the well-known constant

$h = 6.56\ 10^{-27}$.